International Refereed Journal of Engineering and Science (IRJES)
ISSN (Online) 2319-183X, (Print) 2319-1821
Volume 3, Issue 4(April 2014), PP.01-07

A computational experiment in a heuristic for the Fixed Charge Transportation Problem

Gustavo Valentim Loch[1], Arinei Carlos Lindbeck da Silva[2]
[1](Universidade Federal do Paraná, Brazil)
[2](Universidade Federal do Paraná, Brazil)

Abstract:- The Fixed Charge Transportation Problem is a NP-Hard special case of mathematical programming and may be applied to a variety of real life problems. Not only is it used in commodity transportation problems, but also in inventory control, employment scheduling, facility allocation and other applications. While the classic transportation problem involves only variable costs, the Fixed Charge Transportation Problem includes a fixed cost associated with the use of each arc. The fixed cost may represent, for example, tools in Highways or costs to build a road. The objective is then to determine not only the amounts shipped from each source to each destination, but also the arcs to be used. In this paper, we present a heuristic algorithm, comparing the quality solution and computational time with the widely used solver cplex. The tests include problems of different sizes from *5x5* to *25x25* and different ranges of fixed costs.

Keywords:- Computational experiment, Fixed Charge Transportation Problem, Heuristic, MODI

I. INTRODUCTION

The Fixed Charge Transportation Problem (FCTP) arises as a generalization of the Transportation Problem (TP) and a particular case of Fixed Charge Problem (FCP) formulated by Hirsch and Dantzig [1]. The objective is to minimize the total cost for sending a single commodity from m origins to n destinations subject to offers and demands constraints. Its mathematical formulation is described as

$$\min = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + f_{ij}y_{ij} \quad (1)$$

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, \dots, m \quad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, \dots, n \quad (3)$$

$$x_{ij} \le m_{ij}y_{ij}, i = 1, \dots, m; j = 1, \dots, n \quad (4)$$

$$y_{ij} \in \{0,1\} \quad (5)$$

$$x_{ij} \ge 0, i = 1, \dots, m; j = 1, \dots, n \quad (6)$$

The unitary transportation cost is defined by c_{ij} and the fixed cost to activate the route from source i to destination j is defined by f_{ij}. The decision variables x_{ij} and y_{ij} describe the amount to be shipped from origin i to destination j and if the route from i to j is used, respectively. The quantity offered by the source i and demanded by destination j are represented respectively by a_i and b_j. The set of constraints (4) indicates that to transport any quantity greater than zero from source i to destination j it is necessary to activate the route, i.e., $y_{ij}=1$. Additionally, $m_{ij}=\min\{a_i, b_j\}$ indicates the amount that is possible to be shipped if the route is activated.

The set of constraints (2) and (3) is composed of *m+n* equations and there is exactly one redundant constraint [2]. When any one of those constraints is dropped, the remaining system of equation is linearly independent. Therefore, an extreme point is represented by *m+n-1* basic variables.

While it is possible to solve the TP in a polynomial time, Hirsch e Dantzig [1] showed that the FCTP is a NP-Hard and Klose [3] demonstrated it is a NP-Hard even with a single destination. Hirsch and Dantzig [1] and Charnes and Cooper [4] demonstrated that the FCTP admits an optimal solution in an extreme point.

Applications of FCTP are not restricted to distribution problems, but also in different real-life problems. The FCTP was used in a problem of launch vehicles allocation to space missions [5], process selection [1], nuclear plant location [6], solid-waste management [7], wastewater systems [8] and teacher assignment [9].

During the last decades, different heuristic algorithms were proposed to solve the FCTP, such as the ones presented by Balisnki [10], Kuhn and Baumol [11], Cooper and Drebes [12], Sun [13], Adlakha and Kowalski [14][15], Glover [17].

This paper aims to present a simple heuristic for the FCTP, showing that its use in small problems leads to a near optimal value solution in an acceptable computational time.

II. THE MODI METHOD FOR SOLVING TRANSPORTATION PROBLEM

The methods to solve a transportation problem start with an initial Basic Feasible Solution (BFS) followed by an iterative improvement procedure. The BFS may be obtained by different methods, such as the well-known northwest corner, minimal cost, Vogel or Russel. For solution improvement, the most referenced method is the MODI.

The MODI method to solve a TP is highly described in the literature, such as Murty [2] and Hasan [16] and is well known for those having worked with mathematical programming. The method may be resumed in algorithm 1:

Step1. Obtain an initial basic feasible solution.

Step2. Compute the potential (or multiplier) of each row and column. Using the fact that $c_{ij}=u_i-v_j$ for each basic variable, set $u_1=0$ and, recursively, compute the remaining u_i*'s* and v_j*'s*.

Step3. For each non-basic variable, compute $k_{ij}=c_{ij}-u_i-v_j$.

Step4. If there is a negative k_{ij}, chose the variable x_{pq} associated with the most negative k_{pq} to entry the basis. Otherwise, go to step9.

Step5. Find a *θ-loop* in the set of cells consisted by the cell (p,q) and the basic cells.

Step6. Place an entry of $+\theta$ in the cell (p,q) and alternately the entries of $-\theta$ and $+\theta$ among the cells in the θ-loop. The cells place with $-\theta$ are called donor cell and the ones placed with $+\theta$ are called recipient.

Step7. Identify the donor cell (r,s) with the smallest value (in case of a draw, choose one arbitrarily) and set $\theta=x_{rs}$.

Step8. Compute $x_{ij}=x_{ij}+\theta$ for the recipient cells in the *θ-loop* and $x_{ij}=x_{ij}-\theta$ for the donor cell in the θ-loop. Then cell (r,s) becomes non basic and cell (p,q) becomes basic. Go to Step2.

Step9. Finish. The current solution is optimal.

III. THE MODI METHOD ADAPTED FOR FIXED CHARGE TRANSPORTATION PROBLEM

The ideas of the MODI method may also be used to improve a solution of the FCTP. By adapting the algorithm 1 to the FCTP, it is possible to obtain algorithm 2:

Step1. Obtain an initial basic feasible solution.

Step2. Compute the potential (or multiplier) of each row and column. Using the fact that $c_{ij}=u_i-v_j$ for each basic variable, set $u_1=0$ and, recursively, compute the remaining u_i's and v_j*'s*.

Step3. For each non-basic variable:

- compute $k_{ij}=u_i-v_j$
- Find a *θ-loop* in the set of cells consisted by the cell (i,j) and the basic cells
- Place an entry of $+\theta$ in the cell (p,q) and alternately the entries of $-\theta$ and $+\theta$ among the cells in the *θ-loop*. The cells placed with $-\theta$ are called donor cell and the ones placed with $+\theta$ are called recipients.

- Identify the donor cell (r,s) with the smallest value (in case of a draw, choose one arbitrarily) and set $\theta=x_{rs}$
- Compute $_{ij}= \theta k_{ij}+f_{ij}-f_{rs}+D$. D represents a value to correct the effect of degeneracy in the objective function.

Step4. If there is a negative $_{ij}$, chose the variable x_{pq} associated with the most negative $_{pq}$ to enter the basis. Otherwise, go to step9.

Step5. Find a θ*-loop* in the set of cells consisted by the cell (p,q) and the basic cells.

Step6. Place an entry of θ in the cell (p,q) and alternately the entries of $-\theta$ and $+\theta$ among the cells in the θ*-loop* . The cells placed with $-\theta$ are called donor cell and the ones placed with $+\theta$ are called recipients.

Step7. Identify the donor cell (r,s) with the smallest value (in case of a draw, choose one arbitrarily) and set $\theta=x_{rs}$.

Step8. Compute $x_{ij}=xij+\theta$ for the recipient cells in the θ*-loop* and $x_{ij}=x_{ij}-\theta$ for the donor cell in the θ-loop. Then cell (r,s) becomes non basic and cell (p,q) becomes basic. Go to Step4.

Step9. Finish.

The difference between algorithms 1 and 2 lies on the computation of the solution improvement, represented by step3. When there is no fixed charge, it is only necessary to know if the marginal cost k_{ij} of each non-basic variable is negative. On the other hand, when there is a fixed charge, the improvement of variable cost may be less than the extra fixed cost caused by the entry variable.

In the FCTP, there is an optimal solution at an extreme point, but there is no guaranty that a local minimum is a global minimum. So the solution found after algorithm 2 is applied may not, and usually will not, be the global optimal.

For example, consider the problem presented in Table 1.

Table 1 – Variable costs, Fixed costs, offers and demands in a FCTP example

Variable costs c_{ij}						**Fixed costs f_{ij}**						**a_i**		**b_j**
7	3	4	4	7		24	25	7	26	17		77		74
5	5	5	7	4		24	47	48	9	31		170		133
3	6	3	6	5		46	27	41	11	17		130		13
3	4	7	7	5		34	34	30	46	40		14		186
7	7	4	7	5		5	7	28	12	31		180		165
(a)						(b)						(c)		(d)

An initial basic feasible solution for the problem may be the one presented in Table 2.

Tables 2 – Example of initial basic feasible solution for the example from Table 1

x_{ij}	**1**	**2**	**3**	**4**	**5**
1		77			
2		5			165
3	74	51	5		
4				14	
5			8	172	

After that, algorithm 2 is used and the solution presented in Table 3 is obtained.

Table 3 – Solution obtained after algorithm 2 is applied to the solution of Table 2

x_{ij}	**1**	**2**	**3**	**4**	**5**
1		77			
2				5	165
3	74	56	0		
4				14	
5			13	167	

As shown on Table 4, there is no negative $_{ij}$, so the solution is the local minimum.

Table 4 – Values of $_{ij}$ after algorithm 2

$_{ij}=\theta k_{ij}+f_{ij}-f_{rs}+D$	1	2	3	4	5
1	496		7	26	17
2	61	69	44		
3				11	17
4	34	8	41		8
5	57	20			165

In this solution (Table 3), the objective function is $z=3027$, but the optimal value for this problem is $z^*=2964$.

If we force the variable $(4,1)$ to enter the basis, the new objective value will be $z=3027+{}_{41}=3061$. After that, there will be negatives $_{ij}$'s, with $_{34}$=36 being the most negative. After this iteration, the most negative will be $_{42}$=28 and following the next two iterations, $_{52}=20$ and $_{53}=13$, respectively. After these iterations, the objective function reached is $z=2964$, which, coincidently, is the optimal for the problem. The idea of forcing a non-basic variable is that $_{ij}$ is greater than zero to enter the basis and then apply the improvement procedure is used for the proposed algorithm.

IV. THE HEURISTIC ALGORITHM PROPOSED FOR FCTP

In this paper, we propose algorithm 3 as a heuristic algorithm for the FCTP. The first advantage of algorithm 3 is in its similarity to algorithm 1, making it easy for those familiar with the well-known MODI method to understand and use the proposed heuristic. Additionally, the next session shows that the algorithm also provides good results for the problems tested.

Step1. Obtain an initial basic feasible solution. Set $z^*=z$. Set $flag=0$.

Step2. Compute the potential (or multiplier) of each row and column. Using the fact that $c_{ij}=u_i-v_j$ for each basic variable, set $u_1=0$ and, recursively, compute the remaining u_i's and v_j's.

Step3. For each non-basic variable:

- compute $k_{ij}=u_i-v_j$
- Find a θ-*loop* in the set of cells consisted by the cell (i,j) and the basic cells
- Place an entry of $+\theta$ in the cell (p,q) and alternately the entries of $-\theta$ and $+\theta$ among the cells in the θ-*loop*. The cells place with $-\theta$ are called donor cells and the ones placed with $+\theta$ are called recipients.
- Identify the donor cell (r,s) with the smallest value (in case of a draw, choose one arbitrarily) and set $\theta=x_{rs}$
- Compute $_{ij}=\theta k_{ij}+f_{ij}-f_{rs}+D$. D represents a value to correct the effect of degeneracy in the objective function.

Step4. If there is a negative $_{ij}$, chose the variable x_{pq} associated with the most negative $_{pq}$ to enter the basis. Otherwise, if $flag=0$, go to step9 and if $flag=1$, go to step10.

Step5. Find a θ-*loop* in the set of cells consisted by the cell (p,q) and the basic cells.

Step6. Place an entry of θ in the cell (p,q) and alternately the entries of $-\theta$ and $+\theta$ among the cells in the θ-*loop* . The cells placed with $-\theta$ are called donor cells and the ones placed with $+\theta$ are called recipients.

Step7. Identify the donor cell (r,s) with the smallest value (in case of a draw, choose one arbitrarily) and set $\theta=x_{rs}$.

Step8. Compute $x_{ij}=x_{ij}+\theta$ for the recipient cells in the θ-*loop* and $x_{ij}=x_{ij}-\theta$ for the donor cell in the θ-loop. Then cell (r,s) becomes non basic and cell (p,q) becomes basic. Set $z=z+\Delta_{ij}$. If $z< z^*$, set $z^*=z$ and $x^*=x$. Go to Step4.

Step9. Let N be the set of non-basic variables. Set $h=1$ and $flag=1$.

Step10. If $z< z^*$ then set $z^*=z$ and $x^*=x$. If $h<m+n$ then force the h^{th} element of N to enter the basis, set $h=h+1$ and repeat steps 2 to 8.

Step9. Finish.

V. RESULTS

In order to evaluate the algorithm performance, the gap between the optimal solution obtained by the use of cplex and the computational time was analyzed. The considered problems sizes were *5x5*, *10x10*, *15x15*, *20x20* and *25x25*. The offers and demands were randomly defined between *5* and *195*. The variable cost range ranged from *3* to *8*. For the fixed costs, we considered three classes, in which the ranges of fixed costs varied from *5* to *50*, from *5* to *500* and from *5* to *5000*, respectively.

The objective function value obtained by the heuristic, the cplex optimal value, the heuristic time and cplex time are represented by z_h, z_c, t_h and t_c, respectively. For each parameter combination, 200 randomly generated examples were tested and the average results are shown in Table 5.

Table 5 – Solution quality and computational time analysis.

	Fixed costs f_{ij} range					
Problem size	[5,50]		[5,500]		[5,5000]	
	$(z_h-z_c)/z_c$	t_h/t_c	$(z_h-z_c)/z_c$	t_h/t_c	$(z_h-z_c)/z_c$	t_h/t_c
5x5	0,0355%	3,3936%	0,6939%	3,1740%	2,1468%	3,0408%
10x10	0,2221%	12,6729%	1,1939%	24,3978%	4,9921%	19,9253%
15x15	0,4034%	20,4667%	2,7925%	23,7345%	9,3966%	28,4382%
20x20	0,6231%	7,6388%	3,7343%	15,2270%	9,9595%	20,7852%
25x25	0,8001%	7,4629%	4,7553%	13,1057%	9,2788%	23,6588%

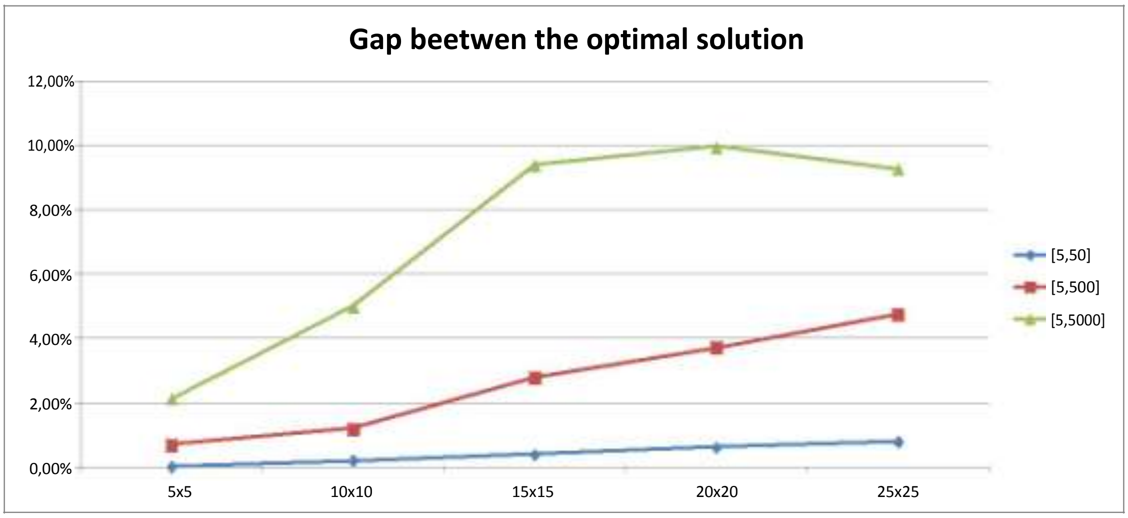

Figure 1 – Comparison of optimal solution gap for different problem sizes

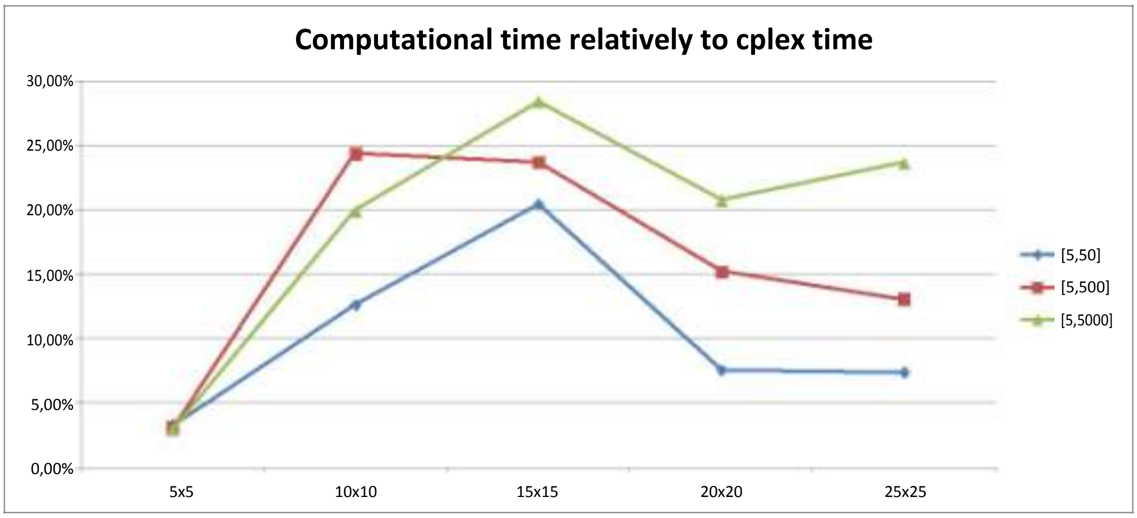

Figure 2 – Comparison of computational time for different problem sizes

On the one hand, it is possible to infer that the gap between the heuristic solution and the optimal solution increases for larger problems and larger ranges of fixed costs. On the other hand, when we analyze the heuristic behavior for small problems and smaller ranges of fixed costs, the results are near optimal.

Another interesting performance indicator is the relative number of times that the optimal solution is reached. In this case, it is possible to infer that for 5x5 conditions, the heuristic provides an optimal solution for the majority of the examples (Table 6).

Table 6 – Number of times that the optimal solution was reached by the heuristic

	Fixed costs range		
Problem size	[5,50]	[5,500]	[5,5000]
5x5	92%	72%	68%
10x10	46%	21%	22%
15x15	12%	4%	2%
20x20	2%	1%	0%
25x25	0%	0%	0%

Since the FCTP is a NP-Hard, it was expected that, for larger problems, the gap between the optimal value would increase and the number of times that the optimal is reached would decrease.

VI. CONCLUSION

The presented heuristic provided good solutions for small problems and, even when the optimal was not achieved, the solutions were near optimal values. In additional to that, computational times were much better than the ones achieved with the benchmark cplex.

For larger problems, the gap between the optimal value was bigger, but the computational time was, in average, less than one quarter of the cplex solution time. It is important to have fast algorithms, even if they are heuristics, because if the FCTP arises as a sub problem that needs to be solved several times in a real life application, the time may be a determinant to the project´s success. Academically, it is important to provide similar methods for different problems. In addition, preliminary studies are made easier, helping to better understand the problem´s characteristics.

The results also showed that good performances for small problem do not necessarily guarantee good solutions for larger problems in the case of the FCTP. Therefore, when a heuristic is proposed for the FCTP it is necessary computational experiments in large problems and not only on the small ones.

REFERENCES

[1] Hirsch, W. M., Dantzig, G. B., The fixed charge problem Naval Research Logistics Quarterly, 15, 1968, 413-424
[2] Murty, K. Linear Programming, Wiley, 1983.
[3] Klose, A. Algorithms for solving the single-sink fixed-charge transportation problem, Computers & Operational Research, 35(6), 2008, 2079-2092.
[4] Charnes, A., Cooper, W. W., Nonlinear Power of Adjacent Extreme Point Methods in Linear Programming. Econometrica, 25, 1957, 132-153.
[5] Stroup, J. W. Allocation of Launch Vehicles to Space Missions: A Fixed-Cost Transportation Problem. Operations Research, 15(6), 1967, 1157-1163.
[6] Dutton, R., Hinman, G., Millham, C. B. The Optimal Location of Nuclear Power Facilities in the Pacific Northwest. Operations Research, 22, 2974, 478-487.
[7] Walker, W. E. A Heuristic Adjacent Extreme Point Algorithm for the Fixed Charge Problem. Management Science, 22, 1976, 587-596.
[8] Jarvis, J. J., Unger, V. E., Rardin, R. L., Moore, R. W., Optimal Design of Regional Wastewater Systems: A Fixed Charge Network Flow Model. Operations Research, 26, 1978, 538-550.
[9] Hultberg, T. H., Cardoso, D. M., The teacher assignment problem: A special case of the fixed charge transportation problem, European Journal of Operational Research 101, 1997, 463-473.
[10] Balinski, M. L., Fixed-cost transportation problem, Naval Research Logistics Quarterly, 8, 1961, 41-54.
[11] Kuhn, H., Baumol, W., An Approximate Algorithm for the Fixed Charge Transportation Problem, Naval Research Logistics Quarterly, 9, 1962, 1-15.
[12] Cooper, L., Drebes, C., An approximate solution method for the fixed charge problem. Naval Research Logistics Quarterly, 14(1), 1967, 101-113.
[13] Sun, M., Aronson, J. E., McKeown, P. G. and Drinka, D. A tabu search heuristic procedure for the fixed charge transportation problem, European Journal of Operational Research, 106, 1998, 441-456.

[14] Adlakha, V. and Kowalski, K., A simple heuristic for solving small fixed-charge transportation problems, Omega: International Journal of Management Science, 31, 2003, 205-211.

[15] Adlakha, V. and Kowalski, K., On the fixed-charge transportation problem. Omega: International Journal of Management Science , 27, 1999, 381-388.

[16] Hasan, M. K., Direct Methods for Finding Optimal Solution of a Transportation Problem are not Always Reliable, International Journal of Engineering and Science, 1(2), 2012, 46-52.

[17] Glover, F., Parametric Ghost Image Processes for Fixed-Charge Problems: A Study of Transportation Networks, Journal of Heuristics, 11(4), 2005, 307-336.